Checking Your Work in MA 16020

April 29, 2016

1. (Integration) To check whether or not you integrated correctly, take the derivative of your answer; it should equal the original function. This will **not** let you know if you forgot the +C (or solved for C incorrectly), but it will check your integration.

WRONG $\int \ln x dx = \frac{1}{x} + C$

CHECK $\frac{d}{dx}[\frac{1}{x} + C] = -x^{-2} \neq \ln x$, so this is wrong.

RIGHT To find $\int \ln x dx$, use integration by parts. ($u = \ln x, dv = dx, du = \frac{1}{x}dx, v = x$)
So $\int \ln x dx = x \ln x - \int x \frac{1}{x} dx = x \ln x - x + C$

CHECK $\frac{d}{dx}[x \ln x - x + C] = \ln x + x(\frac{1}{x}) - 1 = \ln x$, so this is correct!

2. (Differential Equations) To check that you solved a differential equation correctly, take the derivative and plug both y and $\frac{dy}{dt}$ into the left hand side of the original differential equation. It should equal the right hand side when you simplify. Again, this will **not** let you know if you forgot the +C (or solved for C incorrectly).

WRONG We can solve $\dfrac{dy}{dt} + \dfrac{1}{t}y = 3\sqrt{t^2+1}$ using an integrating factor, $u(t) = e^{\int \frac{1}{t} dt} = e^{\ln t} = t$.
Then $u(t)y = \int u(t)Q(t)dt$, and plugging in we get that

$$\begin{aligned}
ty &= \int t(3\sqrt{t^2+1})dt \qquad & u &= t^2+1 \\
& & du &= t dt \qquad \text{(this is wrong, it should be } du = 2tdt) \\
&= \int 3u^{1/2} du \\
&= 2u^{3/2} + C \\
&= 2(t^2+1)^{3/2} + C \\
y &= \frac{2(t^2+1)^{3/2} + C}{t} \\
&= 2t^{-1}(t^2+1)^{3/2} + Ct^{-1}
\end{aligned}$$

CHECK $\dfrac{dy}{dt} = \dfrac{-2(t^2+1)^{3/2}}{t^2} + 6(t^2+1)^{1/2} - \dfrac{C}{t^2}$ and $y = \dfrac{2(t^2+1)^{3/2} + C}{t}$.
We plug these into the original differential equation, $\dfrac{dy}{dt} + \dfrac{1}{t}y = 3\sqrt{t^2+1}$:

$$\frac{-2(t^2+1)^{3/2}}{t^2}+6(t^2+1)^{1/2}-\frac{C}{t^2}+\frac{1}{t}\left(\frac{(t^2+1)^{3/2}+C}{t}\right)=\frac{-(t^2+1)^{3/2}}{t^2}+6(t^2+1)^{1/2}-\frac{C}{t^2},$$

which is NOT equal to $3\sqrt{t^2+1}$ (for instance, plug in $t=1$).

RIGHT

$$\begin{aligned} ty &= \int t(3\sqrt{t^2+1})dt \qquad u = t^2+1 \\ & \qquad\qquad\qquad\qquad\quad du = 2tdt \quad \text{(we fixed our mistake!)} \\ &= \int \frac{3}{2}u^{1/2}du \\ &= u^{3/2}+C \\ &= (t^2+1)^{3/2}+C \\ y &= \frac{(t^2+1)^{3/2}+C}{t} \\ &= t^{-1}(t^2+1)^{3/2}+Ct^{-1} \end{aligned}$$

CHECK $\frac{dy}{dt}=\frac{-(t^2+1)^{3/2}}{t^2}+3(t^2+1)^{1/2}-\frac{C}{t^2}$ and $y=\frac{(t^2+1)^{3/2}+C}{t}$.

We plug these into the original differential equation, $\frac{dy}{dt}+\frac{1}{t}y=3\sqrt{t^2+1}$:

$\frac{-(t^2+1)^{3/2}}{t^2}+3(t^2+1)^{1/2}-\frac{C}{t^2}+\frac{1}{t}\left(\frac{(t^2+1)^{3/2}+C}{t}\right)=3\sqrt{t^2+1}$. So we have the correct solution.

3. (SYSTEM OF EQUATIONS) To check that you found the solution to a system of equations correctly, substitute your solution back in to the system of equations. It should be a solution to each of the equations. (This can be useful when you find the critical points for relative maximum and minimum questions.)

WRONG Solve the following system of equations:

$$\begin{array}{rcrcl} x & + & 2y & = & 3 \\ 2x & - & 2y & = & 0 \end{array}$$

From the first equation, we (incorrectly) get $x=2y+3$.
Substituting into the second equation, we get $2(2y+3)-2y=0$.
Solving, we get that $y=-3$.
Then $x=2(-3)+3=-3$.

CHECK Substituting $x=-3$ and $y=-3$ into the first equation, we get $(-3)+2(-3)=-9$, which is not equal to 3. So $(-3,-3)$ is NOT a solution.

RIGHT From the first equation, we get $x=-2y+3$.
Substituting into the second equation, we get $2(-2y+3)-2y=0$.

Solving, we get that $y = 1$.
Then $x = -2(1) + 3 = 1$.

CHECK Substituting $x = 1$ and $y = 1$ into the first equation, we get $(1) + 2(1) = 3$.
Substituting $x = 1$ and $y = 1$ into the second equation, we get $2(1) - 2(1) = 0$.
So this is a solution.

4. (Reduced Row Echelon Form) To check that you row reduced an augmented matrix $[A|B]$ into *reduced* row echelon form $[I|C]$ form correctly, see if $AC = B$.

WRONG

$$\left[\begin{array}{cc|c} 2 & 2 & 3 \\ 2 & -2 & 0 \end{array}\right] \xrightarrow{-R_1+R_2 \to R_2} \left[\begin{array}{cc|c} 2 & 2 & 3 \\ 0 & -4 & 0 \end{array}\right] \xrightarrow{(-1/4)R_2} \left[\begin{array}{cc|c} 2 & 2 & 3 \\ 0 & 1 & 0 \end{array}\right] \xrightarrow{-2R_2+R_1 \to R_1} \left[\begin{array}{cc|c} 2 & 0 & 3 \\ 0 & 1 & 0 \end{array}\right] \xrightarrow{(1/2)R_1} \left[\begin{array}{cc|c} 1 & 0 & 3/2 \\ 0 & 1 & 0 \end{array}\right]$$

CHECK $\begin{bmatrix} 2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 3/2 \\ 0 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \end{bmatrix} \neq \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ So something went wrong.

RIGHT

$$\left[\begin{array}{cc|c} 2 & 2 & 3 \\ 2 & -2 & 0 \end{array}\right] \xrightarrow{-R_1+R_2 \to R_2} \left[\begin{array}{cc|c} 2 & 2 & 3 \\ 0 & -4 & -3 \end{array}\right] \xrightarrow{(-1/4)R_2} \left[\begin{array}{cc|c} 2 & 2 & 3 \\ 0 & 1 & 3/4 \end{array}\right] \xrightarrow{-2R_2+R_1 \to R_1} \left[\begin{array}{cc|c} 2 & 0 & 3/2 \\ 0 & 1 & 3/4 \end{array}\right] \xrightarrow{(1/2)R_1} \left[\begin{array}{cc|c} 1 & 0 & 3/4 \\ 0 & 1 & 3/4 \end{array}\right]$$

CHECK $\begin{bmatrix} 2 & 2 \\ 2 & -2 \end{bmatrix} \begin{bmatrix} 3/4 \\ 3/4 \end{bmatrix} = \begin{bmatrix} 3 \\ 0 \end{bmatrix}$ So we row reduced correctly.

5. (Matrix Inverse) To check that you found the inverse, A^{-1}, of a matrix A correctly, see if $AA^{-1} = I$.
(This is just a special case of (4)).

WRONG We can find the inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by first finding the determinant:

$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2.$

Suppose we write $A^{-1} = \dfrac{1}{-2} \begin{bmatrix} 4 & 2 \\ 3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & -1 \\ -3/2 & -1/2 \end{bmatrix}$ (this is wrong since we forgot to multiply the entries on the off-diagonal by -1).

CHECK $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -2 & -1 \\ -3/2 & -1/2 \end{bmatrix} = \begin{bmatrix} -5 & * \\ & * \end{bmatrix}$

Without even finding the other entries, we can tell this isn't going to be the identity matrix. So something went wrong.

RIGHT We can find the inverse of $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ by first finding the determinant:

$\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = 4 - 6 = -2.$

Now we correctly write $A^{-1} = \dfrac{1}{-2} \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix}.$

CHECK $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} -2 & 1 \\ 3/2 & -1/2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ So we found the inverse correctly.